2024 KCATM
High School Math Contest

Name: _____

Grade: _____

High School: _____

e-mail address: _____

phone number: _____

INSTRUCTIONS

The following test consists of 20 questions. Use whatever resources you like to solve these problems. Each question is worth 5 points. Partial credit will be given for making progress. However, you must show your work to get any credit. **PLEASE PRINT THIS DOCUMENT OFF IN LANDSCAPE MODE!**

THE CONTEST and REGISTRATION

- There is no cost for this contest.
- There are two grade categories: 9th and 10th graders will be graded in one category, while 11th and 12th graders will be graded in a second.
- All students will be informed of their score approximately one week after the date below, and the top-3 in each category will be officially recognized.

SUBMIT YOUR TEST AND ANSWERS EITHER …

BY MAIL

Mike Round
13234 Long Street
Overland Park, KS 66213

Entries must be post-marked no later than April 14th.

BY EMAIL

round12345@aol.com

Entries must be received no later than April 14th.

QUESTIONS

High School Test Coordinator: Mike Round
(913) 515-3911 round12345@aol.com

ADDITIONAL THOUGHTS

MAKE SURE YOU ANSWER THE QUESTION ASKED!

PROVIDE AS MUCH EXPLANATION AS POSSIBLE – NOT JUST THE ANSWER!

THERE ARE TWO BONUS PROBLEMS!
These don't apply in the scoring – I was just curious to see how bright students attack problems like these!

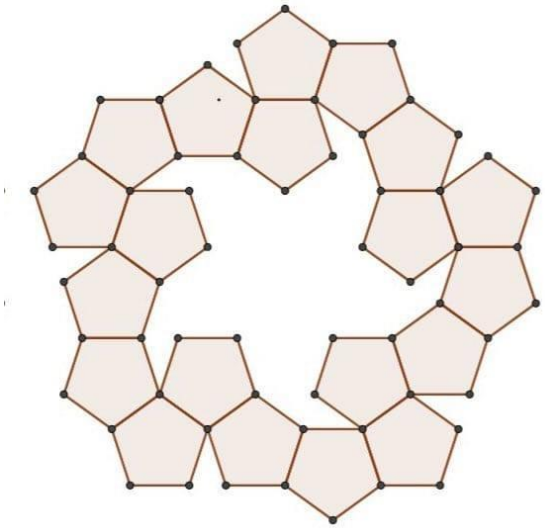
1. What is the area bounded by the following equations in Quadrant #1:

$$x^2 + y^2 < 9$$

$$y > 2x$$

2. I took a picture of a heron recently. Beautiful birds! If I put the letters "HERONS" in a hat and pull five letters out (placing them in order as I go) what are the *odds* I've drawn "HERON"? (Your answer must include the word “to”).

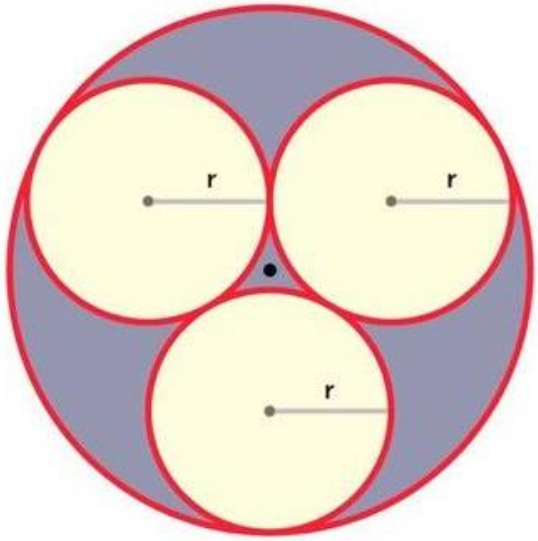
3. The area of each regular pentagon below is 1. What is the area of the inside white figure?



4. In the movie "Unstoppable" two trains are on the same track approaching each other. Triple 7 is unmanned and traveling south at 70 mph. Train 1206 is traveling north at an unknown speed. They are 50 miles apart, and there is a siding 30 miles away from 1206. How fast does 1206 need to go to get to the siding to avoid collision?

5. $x + \frac{1}{x} = 3$. Find x.

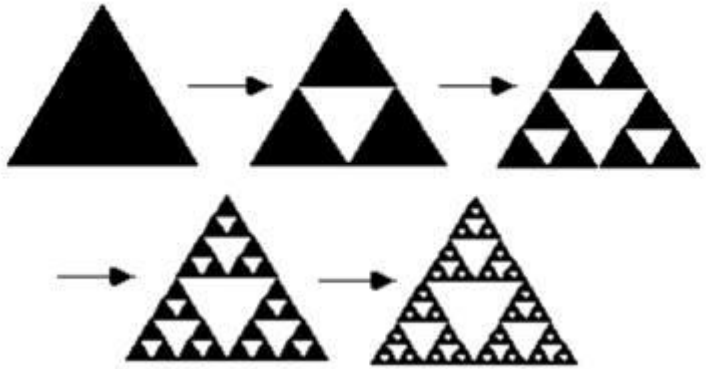
6. Three identical circles fit perfectly into a larger circle, as shown below. If each smaller circle has radius r, what is the area of the shaded region (in terms of *r*)?



7. I make four stops on a bus route (with scheduled departures at 8:00, 8:10, 8:20, and 8:30). After the last stop, it then takes 10 minutes to drive to school. On my first day, I arrived at my first stop right on time. The driving time between the next four stops (three stops and school) can be anywhere from 3 minutes to 20 minutes.

   If the average driving time between these four driving distances is 10 minutes, what is the earliest and latest times I can arrive at school? (Note: the bus cannot leave before any scheduled departure time).

8. I have an equilateral triangle with area 1 square unit. I then remove an equilateral triangle with vertices touching the sides of the original triangle. I repeat the process three more times, each time removing an inner equilateral triangle from every shaded triangle. What is the area of the shaded region in the fifth image below?



9. On the 600 mile drive home from Denver to Kansas City, I drove 80mph for the first 300 miles, but only 60mph for the second 300 miles. What was my average speed for the trip home?

10. A standard pool table has 6 pockets. Suppose the lower left corner pocket has coordinates (0,0) and the upper right pocket has coordinates (250,500). The cue ball is sitting at (50,100), I strike it, it hits the wall furthest from the ball, and ends up in the lower right pocket. What are the coordinates where the cue ball struck the wall?

11. Suppose the high temperature (in degrees Fahrenheit) for Tuesday is half what it was Monday. Monday's high temperature was 76 degrees. What was the % decrease from Monday to Tuesday in degrees Celsius?

12. What is the maximum value of $y = sin(x) + sin(3x)$?

13. To get to a friend's house, I drove 1 mile north, 2 miles east, 2 more miles north, and 2 more miles east. I averaged 40 mph. If instead there was a circular road that started and stopped at my friend's house, how fast would I have to drive to arrive at the same time?

14. You need to make 5 stops. Your starting point is the point (0,0) on the Cartesian Plane. Your five stops are at the following locations:

    (-3, 4) (2, -3) (0, -1) (2, 0) (-4, 0)

    Suppose you get reimbursed based on mileage and you want your travels to be as FAR as possible. What is the LONGEST distance you would travel to get to all five stops (only once)?

15. Suppose I told you a particular year is a leap year. If that's all you knew, what is the probability Easter is in March? (yes, research how Easter is actually calculated.)

16. Consider a population of rabbits on a secluded island. The rate of change of the population size, P, with respect to time, t, is proportional to the square root of P, where $P > 0$. At $t = 0$ the population size was 100 rabbits. Find the population size at 4 years.

17. Mike can mow a lawn by himself in 3 hours. Jack can mow the same lawn by himself in 2 hours. If they work together and stay out of each other's way, how long will it take in minutes for the lawn to be mowed?

18. In a town, there are two types of families:

    - Type A families always have children until they get a boy and then stop having children.
    - Type B families always have children until they have at least one boy and one girl, and then stop.

    Assuming that every child is equally likely to be a boy or a girl, what is the expected number of children in a Type A family and a Type B family?

19. Suppose there is a population of 10,000 people and we estimate 1% have a disease.

    I've invented a test to find who has – and who does not – have the disease. If you have the disease, my test will return a positive reading 95% of the time. If you don't have the disease, my test will return a negative reading 95% of the time.

    If a person is tested and the results come back positive, what is the probability the person actually has the disease?

20. I'm on top of a building 200m tall. I have two baseballs. The first I throw horizontally at 90mph. At the same time I'm throwing the first baseball with my right hand, I'm simply dropping the second baseball from my left hand. After two seconds, how far apart are the two baseballs. (Ignore my height as well as air resistance).

21. BONUS #1: Mike can mow *a*% of lawn by himself in *b* hours. Jack can mow *c*% of the lawn in *d* hours. If they work together and stay out of each other's way, how long will it take in minutes for the lawn to be mowed?

    (Note: $a + c$ DO NOT have to equal 100%).

22. BONUS #2: Suppose there is a population of 10,000 people but we don't have a good estimate of how many people have a disease. All we have is the % of people who have tested positive.

    I've invented a test to find who has – and who does not – have the disease. If you have the disease, my test will return a positive reading 95% of the time. If you don't have the disease, my test will return a negative reading 95% of the time.

    If *a*% of people tested have come testing positive, what Is the estimate *b*% of the people who have the disease?