

2019 KCATM High School Math Contest

Name: _____

Grade: _____

High School: _____

e-mail address: _____

phone number: _____

INSTRUCTIONS

The following test consists of 20 questions. Use whatever resources you like to solve these problems. Each question is worth 5 points. Partial credit will be given for making progress. However, you must show your work to get any credit.

PLEASE PRINT THIS OFF IN LANDSCAPE MODE!

THE CONTEST and REGISTRATION

There is no cost for this contest. 9th and 10th graders will be graded in one category, while 11th and 12th graders will be graded in another. All students will be informed of their score by April 6th, and the top-3 in each grade level will be recognized at the annual banquet.

ALL ENTRIES MUST BE POST-MARKED NO LATER THAN MARCH 31ST.

MAIL YOUR ANSWERS

Mike Round
13234 Long Street
Overland Park, KS 66213

QUESTIONS

High School Test Coordinator: Mike Round
(913) 515-3911 round12345@aol.com

1

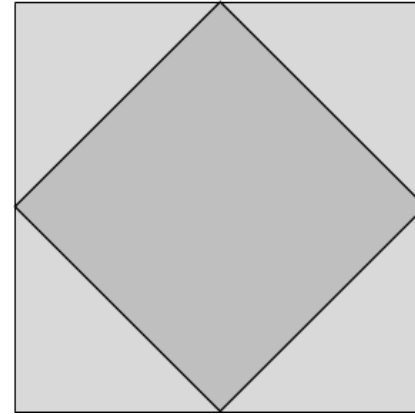
SUMS and DIFFERENCES

Expand $y = (x + y)^2(x - y)^2$.

2

A SQUARE DEAL

An outer square has sides of length 4". A second square is perfectly embedded in the first, as shown below. What is the area of the inner square?



3

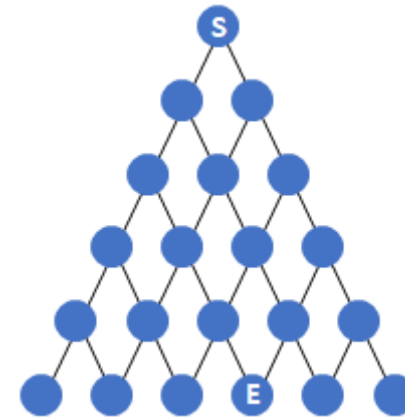
A MATTER OF HEIGHT

The Burj Khalifa in Dubai is the tallest building in the world, at 2,717 feet tall. How long would it take a laser, traveling at the speed of light, to go from ground level to the building's top, and back again?

4

BINARY PATHS

Starting at the "S", you move downward along a line to the next circle, eventually reaching the bottom. How many different paths would lead you to the "E"?



5

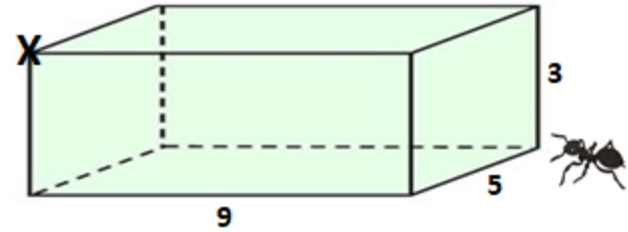
"THE WAVE"

What is the maximum value of $f(t) = |\sin(t) + 2\cos(2t)|$?

6

"X" MARKS THE SPOT

What is the shortest distance the ant, starting at one corner of an empty (but sealed) box, can travel to get to the "X"?



7

SUPPLY and DEMAND

I have a store selling I-Phones. I have six customers per day, and the probability any one of them purchases a phone is 50%. At the beginning of the week, I have in stock the expected number of phone sales for a given day, and each day I replace any sold phones.

At the end of a 5-day work week, what is $p(\text{sales} \geq 15)$?

8

BASKETBALL and the 3-POINT SHOT

Suppose my basketball team currently takes 100 shots per game. 70% of these are worth 2-points each, while the remaining 30% are worth 3-points each. We make 50% of our 2-point attempts, and 35% of our 3-point attempts.

Assuming the made field-goal percentages are the same and we still take 100 shots, how many more points will we score per game if *half* of our attempts are worth 3-points? (Assume the number of free throws made is unchanged).

9
COUPONS

After making a purchase of \$24, I mail in a coupon worth 20% of the purchase price, using a 55¢ stamp. Each subsequent purchase is half of my previous purchase price, and I repeat the coupon process until it does not make financial sense to do so. How much *total* money will I have saved?

10
POOL – WITH A CAPITAL “P”

If a cue ball placed at the coordinates (22, 55) was evenly struck so it hit the 1st wall at the point (0, 88), and bounced off with no spin, what are the coordinates when the ball strikes the 6th wall?



11

THE BEZIER CURVE

Line l connects points $(1,1)$ and $(5,5)$, while line m connects points $(5,5)$ and $(9,0)$. Line n connects the midpoints of lines l and m . What is the midpoint of line n ?

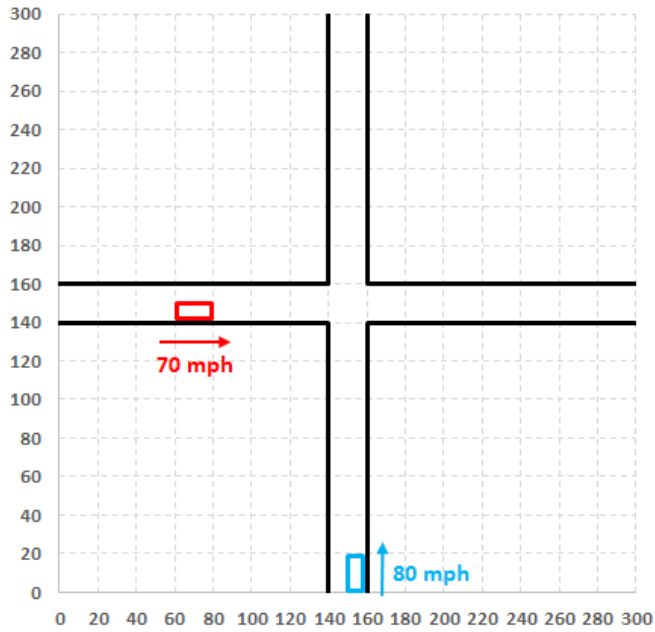
12

"CENTRAL TENDANCY"

There are 10 quiz scores, ranging from 0 to 10. The *median* score is 5, and the most frequent (*mode*) score is also 5. What is the highest possible *mean* score?

13
AVOIDING DISASTER

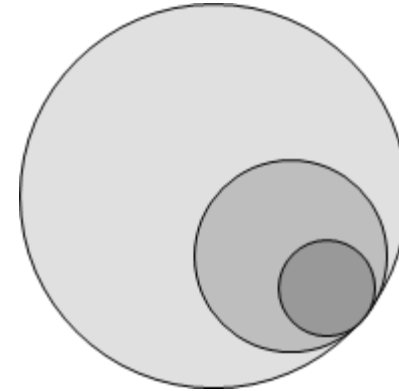
The (x, y) –intersection below is in feet. At the intersection the stop-lights are not working. If the red car is traveling east at 70mph , and the blue car is traveling north at 80mph , will the cars collide in the intersection?



core?

14
A CIRCLE within A CIRCLE within A CIRCLE

The following three circles have radii equal to $4''$, $2''$, and $1''$, and all touch at one point. What is the size of the smallest circle relative to the largest circle?



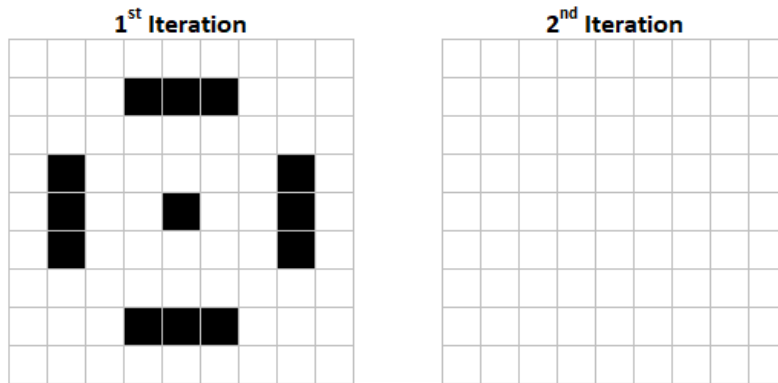
15
THE GAME OF LIFE

John Conway's "Game of Life" simulates population growth. Each cell has four "neighbors" (above, below, right, and left). From one iteration to the next, a "space" can either die, survive, or grow. The rules are as follows:

GAME of LIFE RULES

1. For a space that is currently populated:
 - if it has one or no neighbors, it dies (from solitude);
 - if it has four or more neighbors, it dies (from overpopulation);
 - if it has two or three neighbors, it survives.
2. For a space that is currently 'empty' or 'unpopulated':
 - if it has three neighbors, it becomes populated.

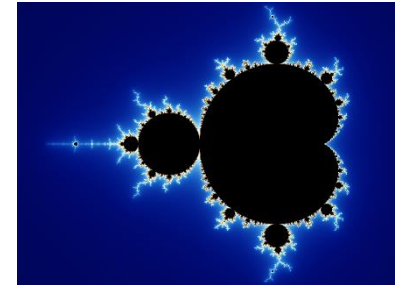
What is the probability the 10 occupied cells in the 1st iteration below will survive in the 2nd iteration?



16
THE MANDELBROT SET

The Mandelbrot Set is generally defined by the equation $Z_{n+1} \leftarrow Z_n^2 + c$.

If $c = 2 + 3i$ and $Z_0 = 1 - 2i$, find Z_1 .

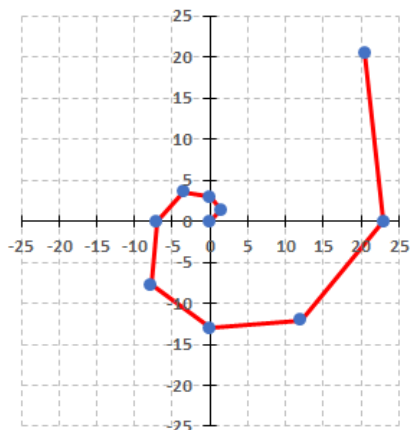


17
A PRIME SPIRAL

Prime numbers are plotted on the following (x, y) –grid, spacing each successive prime 45° from the previous number. For example:

<u>Prime</u>	<u>Degrees</u>
2	45°
3	90°

What are the (x, y) –coordinates of the 9th prime number?



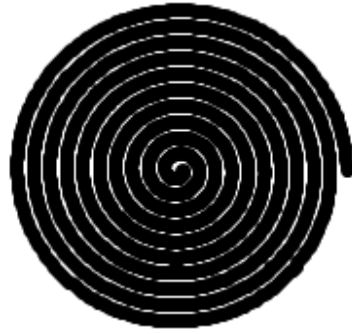
18
“LET THEM EAT CAKE”

Cake batter is made in a circular prism, with diameter 8” and height 4”, and filled to the top. If this batter is then poured into a rectangular-based prism with length 12” and width 9”, how high will the batter reach?

19

A NEW CARPET

When unrolled, this 1" thick carpet will cover approximately what length of my room?



20

A TRIG IDENTITY

Prove the trigonometric formula:

$$\sin(x + y) = \sin(x) \cdot \cos(y) + \cos(x) \cdot \sin(y).$$