KCATM Calculus Test **20 questions**
Calculators Permitted **Multiple Choice**

1. Find $\lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}=$

a) $\tan(x)$ b) $\cos(x)$ c) $\sin(x)$

d) $\sec(x)$ e) $\csc(x)$

2. If $f(x)$ is a continuous function defined by $f(x)=\begin{cases} x^2-bx, & x\le 2 \\ 2\sin\left(\frac{\pi x}{4}\right), & x>2 \end{cases}$,

find the value of b.

a) -2 b) -1 c) 0

d) 1 e) 2

3. The slope of the tangent line to the graph of $f(x)=\frac{kx^3+1}{x-1}$ at $x=2$ is 3, where *k* is a constant. What is the value of *k* ?

a) 1 b) 2 c) 3

d) 4 e) 5

4. A spherical balloon is being inflated at a rate of 10 cubic inches per minute. Find the rate of change of the length of the radius of the balloon, in inches per minute, when the balloon has been inflated to exactly 36π cubic inches. ***Hint: the volume of a sphere is*** $V=\frac{4}{3}\pi r^3$.

a) 0.088 b) 0.117 c) 0.032

d) 0.104 e) 0.163

5. $y=x^2+\sin(2x)$, find $\frac{d^{41}y}{dx^{41}}$.

a) $\cos(2x)$ b) $2x+\cos(2x)$ c) $-\cos(2x)$

d) $2x-\cos(2x)$ e) $2^{41}\cos(2x)$

6. If $k(x)=h(g(x))$, using the graphs of $h(x)$ and $g(x)$ below, evaluate $k'(0)$.

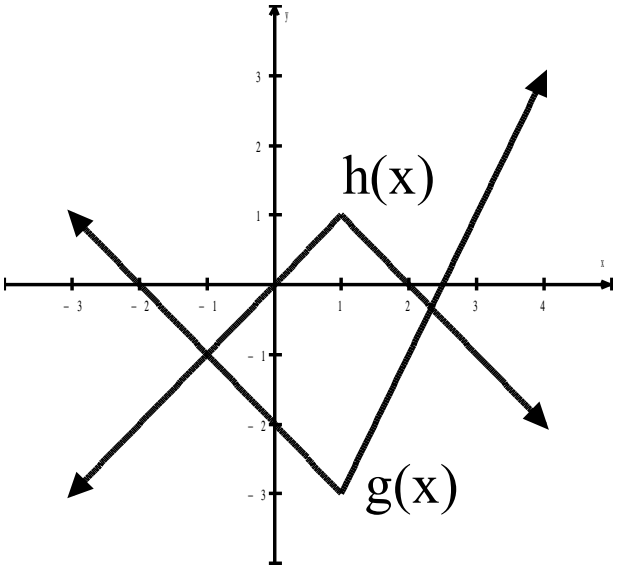
a) -2

b) -1

c) 0

d) 1

e) 2



7. At what value(s) of x does $f(x)=x^3-(3k^2)x$ have a relative minimum?

a) 0 only b) k only c) -k only

d) -k & k only e) -k, 0, & k only

8. Find the average value of $g(x)=\sqrt{x}+1$ on the interval $[0,\ 4]$.

a) $\frac{2}{3}$ b) $\frac{5}{2}$ c) $\frac{6}{5}$

d) $\frac{7}{3}$ e) $\frac{11}{2}$

9. Let R be the region bounded by the x-axis and the curve $y=x^3$ on the interval $[0, 2]$. The vertical line $x=c$ divides R into two regions of equal areas. Find the value of c.

a) $\sqrt[4]{7}$ b) $\sqrt[3]{5}$ c) $\sqrt[3]{6}$

d) $\sqrt[4]{8}$ e) None of These

10. The area of the shaded region below the curve $y=\frac{1}{x}$ and above the x-axis on the interval $[2,k]$, where k is a constant, is $\ln 5$ *(see figure)*. Find the value of k.

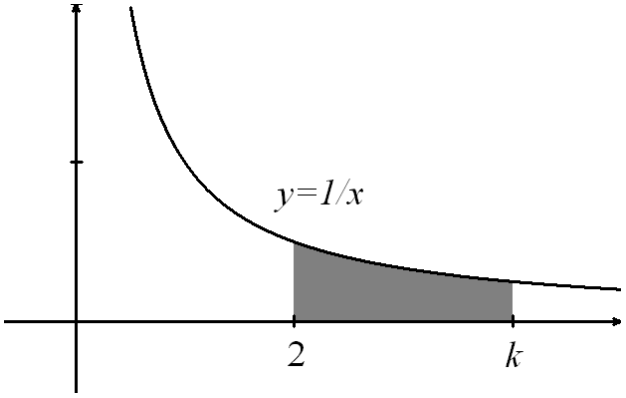
a) 6

b) 8

c) 10

d) 12

e) 14



11. Find $\int \frac{e^{kx}}{(e^{kx}+1)^2}dx$, where k is a constant.

a) $\frac{-1}{k(e^{kx}+1)}+C$ b) $\ln|e^{kx}+1|+C$ c) $\frac{-1}{(e^{kx}+1)^2}+C$

d) $\frac{1}{k}\ln|e^{kx}+1|$ e) $\arcsin e^{kx}+C$

12. Find the area bounded by the the curves $y=x^2$ and $y=\sqrt{x}$.

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{5}$

d) $\frac{1}{12}$ e) $\frac{1}{7}$

13. Find $\int \frac{1}{2\sqrt{x}(k^2+x)}dx$, where k is a constant.

a) $\ln|k^2+x^2|+C$

b) $\frac{1}{k}\arctan\frac{\sqrt{x}}{k}+C$

c) $\frac{-1}{(x^2+k^2)^2}+C$

d) $\arcsin\frac{\sqrt{x}}{k}+C$

e) $\sinh\frac{\sqrt{x}}{k}+C$

14. Selected values of $f(x)$, $g(x)$, and some of their derivatives are given in the table below. Find $\lim_{x\to 0}\frac{f(x)}{g(x)}$.

a) 0

b) 1

c) 2

d) 3

e) 4

x	-1	0	1	2
$f(x)$	0	0	2	2
$f'(x)$	1	0	1	1
$f''(x)$	2	2	4	2
$g(x)$	2	0	3	0
$g'(x)$	2	0	3	0
$g''(x)$	1	1	2	2

15. Find $\int x e^x dx$

a) $x e^x - e^x + C$

b) $x e^x - x + C$

c) $\frac{x^2 e^x}{2}+C$

d) $e^x + x + C$

e) $x e^x + C$

16. Which of the following series converge?

I. $\sum_{n=1}^{\infty} 4\left(\frac{5}{3}\right)^n$

II. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

III. $\sum_{n=1}^{\infty}\frac{1}{n}$

a) I only

b) II only

c) III only

d) I and II only

e) II and III only

17. Find the slope of the parametric curve $\begin{matrix} x(t)=t^2 \\ y(t)=t^3+t \end{matrix}$ at $t=2$.

a) $\frac{13}{4}$ b) $\frac{8}{3}$ c) $\frac{5}{2}$

d) $\frac{21}{6}$ e) $\frac{2}{3}$

18. Which of the following integrals represents the area bounded in the first quadrant by the polar curve $r=f(\theta)$, and the radial lines $\theta=0$ and $\theta=\frac{\pi}{2}$.

a) $\int_0^{\frac{\pi}{2}} [f(\theta)] \ d\theta$ b) $2\int_0^{\frac{\pi}{2}} [f(\theta)] \ d\theta$ c) $\frac{1}{2}\int_0^{\frac{\pi}{2}} [f(\theta)] \ d\theta$

d) $2\int_0^{\frac{\pi}{2}} [f(\theta)]^2 \ d\theta$ e) $\frac{1}{2}\int_0^{\frac{\pi}{2}} [f(\theta)]^2 \ d\theta$

19. Find the unit tangent vector to the curve in space $f(t)=t\,\boldsymbol{i}+t^3\,\boldsymbol{j}+t^2\,\boldsymbol{k}$ at the point (2,8,4).

a) $\frac{1}{\sqrt{196}}\boldsymbol{i}+\frac{12}{\sqrt{196}}\boldsymbol{j}+\frac{4}{\sqrt{196}}\boldsymbol{k}$ b) $\frac{1}{\sqrt{161}}\boldsymbol{i}+\frac{12}{\sqrt{161}}\boldsymbol{j}+\frac{4}{\sqrt{161}}\boldsymbol{k}$

c) $\frac{1}{\sqrt{84}}\boldsymbol{i}+\frac{12}{\sqrt{84}}\boldsymbol{j}+\frac{4}{\sqrt{84}}\boldsymbol{k}$ d) $\frac{1}{\sqrt{40}}\boldsymbol{i}+\frac{12}{\sqrt{40}}\boldsymbol{j}+\frac{4}{\sqrt{40}}\boldsymbol{k}$

e) $\frac{1}{\sqrt{10}}\boldsymbol{i}+\frac{12}{\sqrt{10}}\boldsymbol{j}+\frac{4}{\sqrt{10}}\boldsymbol{k}$

20. Evaluate the integral $\int_C (x+2)ds$, where C is the curve $r(t)=t\,\boldsymbol{i}+\frac{4}{3}t^{(3/2)}\,\boldsymbol{j}+\frac{1}{2}t^2\,\boldsymbol{k}$, $0\le t\le 2$.

a) 11.418 b) 12.654 c)13.665

d) 14.383 e) 15.291